DISCREPANCIES BETWEEN THEORY AND OBSERVATION OF THE MOON'S MOTION.

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In studying recently the question of the moon's irregularities, I discovered an apparent source of error affecting lunar computations, whose explanation will probably interest many scientists, though of more special value to the astronomer.

The problem of the moon s motion has been one of the most interesting, though troublesome, in the series which astronomers have attacked with such success, as one by one the difficulties have been overcome, and the victors received their well-earned honors. Yet it is well known that the differences between the predicted and observed places of the moon are greater than in the case of any other heavenly body, and that these discrepancies have excited the special efforts of investigators.

Thus a comparison of the Washington observations with the American Ephemeris for 1870, indicates errors in the predicted right ascension of the moon, extending from +0.4 s. to —0.7 s.

Among the interesting phenomena dependent upon the motions of the moon, lunar and solar eclipses are conspicuous, and observations of these have naturally been used in the determination of elements or in the verification of theories, and any error pertaining to such observations would serve to introduce error into lunar tables.

An eclipse of the moon occurs when she passes into the earth's shadow, and the axis or central line of the earth's shadow is usually reckoned as the prolongation of the line connecting the center of the sun with the center of the earth. But this supposition contains an appreciable error. For since light requires an appreciable extent of time to pass from the earth to the moon, the earth advances in her orbit during that small interval, so that the part of the shadow found at any distance from the earth, say at the moon, lags a little behind the corresponding place of the earth.

It may easily be shown that the axis of the earth's shadow thus lags by an angle of about 20″ — equal to the angle of aberration — and as the entire cone of shadow tends to lag equally in angular measurement, it will follow that the eclipse will begin and end earlier than if the earth's shadow were projected directly behind itself.

In other words, the central moment of a lunar eclipse does not coincide with the instant of the moon's opposition, but differs by the time required for the moon to gain about 20″ relatively to the earth's

position in her orbit, which may be 40 or 50 seconds of time or more.

In the case of a solar eclipse, some of these relations are reversed, though the result is nearly the same. Considering the motions of the earth and moon with reference to the sun, the moon is first in advance of the earth, while the earth is gaining upon her, but as the moon's shadow lags also, so the earth encounters it earlier, and by nearly the same interval of time, as happened in the case of the lunar eclipse, and the central moment of the solar eclipse occurs before the instant of conjunction.

It follows, then, that a lunar theory which includes the supposition that the central moments of lunar and solar eclipses indicate the instants of opposition and conjunction must involve an appreciable error, which would affect the tables based upon it.

It is noticeable that the error here pointed out—20″ in longitude —corresponding to about 1.3 s. in right ascension, expresses so nearly the range of the discrepancies between the lunar tables and the results of observation.

Whether the elimination of the effects of this error would serve to reconcile completely theory and observation, can only be decided by actual experiment, but it seems reasonable to expect much from such an elimination.

The lagging of the shadows suggests other curious deductions, which may be mentioned:

When the transit of Venus occurs, she will not, in the ordinary sense of the word, be *seen* upon the disc of the sun, but she will intercept certain rays of light, while only the *place* of the intercepted rays is perceived. But the place of these rays, or the shadow, lags, and, since the velocity of Venus is greater than that of the earth, the angle of lagging is greater than in the case of the earth's shadow. It will happen, then, that Venus will be in a line with a tangent to the sun and earth several minutes before an observer on the earth can perceive the indications.

If Venus were able to shine and make her light perceptible while passing in front of the sun, she would be perceived sooner than will actually be the case, and all the circumstances of the transit will appear later on this account.

The angle of aberration due to the velocity of Venus is about 24″, and, since this measures the angle of lagging, the shadow will not overtake the earth until Venus shall have advanced about 24″ past the line of tangency. The hourly motions being respectively +10-.980 s. and 6.267 s. in right ascension, it will require about five minutes to accomplish the passage, and this interval indicates the apparent delay that may be expected.

Again, in the case of the solar eclipse, the absolute velocity of the moon is less than that of the earth, by a difference due to the velocity of the moon about the earth, which may be determined with a good degree of accuracy. The angle of lagging will, therefore, be less, and the time occupied by the earth in gaining it will also be less, so that the change in the angle of aberration, due to the velocity of the moon about the earth, will be indicated on a magnified scale.

The ratio of this change to the entire angle of aberration would again indicate the absolute velocity of light, knowing which, and the time occupied by light in coming to us from the sun, the distance of the sun becomes determinable.

The value of this method of determining the solar parallax will depend upon the precision of the lunar tables; but, with the elimination of the effects of the error above explained, it is possible such precision may be attained as shall make the method valuable.

The comparative frequency of eclipses would enable astronomers to accumulate observations rapidly, and the more rapidly reduce the degree of error.

POSTSCRIPT.

The foregoing remains the same as originally prepared, though from further inquiry it appears that in the case of the lunar and solar eclipses *apparent* instead of *true* geocentric opposition and conjunction are understood in the elements given in the Nautical Almanac for those computations, so that no error in the moon's place could thus occur. A similar remark applies in the case of the transit of Venus.

The fact of the lagging of planetary shadows seems, however, to deserve recognition, though it furnishes no aid in the attempt to diminish the discrepancies between lunar theory and observation. And though these discrepancies are still so great as to indicate a specific cause which has so far eluded discovery, it is yet reasonable to believe that the true cause lies within the operations of known law; that this cause will ultimately be found out, when the moon will appear to move completely subject to the law of gravitation, and will conform gracefully to the predictions of astronomers.

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